

# Computable General Equilibrium (CGE) Models: A Short Course

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## Session Three: IMPLEMENTATION

# Session 3: Implementation

- ◆ Database
- ◆ Calibration
- ◆ Solution Strategy
- ◆ GAMS Software

# Database

- ◆ To apply a CGE requires an empirical specification of all equations
- ◆ Many equations, many parameters to determine
- ◆ Often a mathematical procedure where only a single “observation” is used to estimate parameters

# SAM, NIPA, IO

- ◆ Required data
  - IO table, might need aggregation
  - NIPA
  - May need census data
  - Estimates of sectoral capital stock & depreciation
- ◆ Generate a SAM
- ◆ Capital Composition Matrix

# Level of aggregation

- ◆ No set of rules on the appropriate level of aggregation; judgmental, and depends on:
  - Focus of the model
  - Availability of data
  - Feasibility of computation
  - Time constraints.

# Census data

- ◆ May need census data for:
  - Population
  - Sectoral employment
  - Categories of labor
  - Household types

# Sectoral Capital Stock

- ◆ A reasonable estimate of sectoral capital stocks is indispensable in any economic model that deals with sectoral production.
- ◆ Methods:
  - Use historical investment data, adjusted for price inflation and depreciation, to obtain capital stocks.
  - Use capital-output ratios to derive rough estimates of capital stocks.
  - Derive an estimate of capital stock using data on capital income and net rate of return to capital

# Sectoral Depreciation Rates

- ◆ Use independent estimates of sectoral depreciation rates if exists
- ◆ Otherwise, use the available historical data on sectoral capital formation estimate these rates by determining only the depreciation rates of capital types that are present in the economy.

# SAM

- ◆ Main data structure underlying CGEs
- ◆ An internally consistent accounting framework
- ◆ A snapshot of the economy at a certain time
- ◆ Used to calibrate the CGE model to the base year data by determining key parameters
- ◆ Construction closely related to the modeling framework and policy questions

# Constructing a SAM

- ◆ A SAM plays a very crucial role in the outcome of a CGE model
- ◆ As important as the final SAM is the procedure that generates it
- ◆ The quality of data and assumptions made directly affect the results of the CGE model
- ◆ Either input-output tables or national income and product accounts can be used as a starting basis to generate a SAM

# A National SNA SAM

Expenditures					
Receipts	Activity	Commodity	Factors	Institutions	World
<b>Activity</b>		Domestic Supply			Exports
<b>Commodity</b>	Intermediate inputs			Final demand	
<b>Factors</b>	Value added				
<b>Institutions</b>	Indirect taxes	Indirect taxes and tariffs	Factor income	Transfers And taxes	Inflows
<b>World</b>		Imports			
<b>Totals</b>	Total costs	Total supply	Total factor income	Domestic income	Foreign Exchange inflow

# Capital Composition Matrix

- ◆ Coefficients describing the make up of sectoral capital stock (not in SAM)
- ◆ Put in a matrix form “Capital Composition Matrix”
- ◆ Each element indicates how much of the capital goods produced by the row sector contributes to make up one unit of capital in the column sector, thus each column of the matrix adds up to one.
- ◆ The sector that produces capital is called the *sector of origin* and the sector that purchases capital is referred to as the *sector of destination*

# Calibration

- ◆ Econometric estimation of a large number of parameters in a CGE model is a major task even when the extensive time series data are available
- ◆ With a few exceptions, CGE models determine parameters mathematically from a social accounting matrix

# Calibration Procedure

- ◆ Determines the set of parameters and exogenous variables so that the CGE model will replicate the economy represented in the SAM
- ◆ It is common to interpret flow entries in the SAM as a measure of both real and nominal magnitude by choosing a proper unit for all physical quantities so that the prices equal one.

# Solution Strategy

- ◆ In the 1970s, major advances were made in solution techniques that permitted the implementation of CGE models.
- ◆ There are four different approaches:
  - Fixed point
  - System of non-linear equations
  - Linearization
  - Non-linear programming

# Fixed point

- ◆ The solution of a CGE model can be formulated as a problem in finding a fixed point in a mapping of prices to prices through excess demand equations.
- ◆ Pioneered by Herbert Scarf
- ◆ E.g. Scarf and Hansen. 1973. The computation of economic equilibria. Yale Univ. Press.

# System of non-linear equations

- ◆ Treat CGE model as a collection of non-linear algebraic equations and solve directly with numerical solution techniques.
- ◆ Within this framework, there are variations depending on how to solve the system.
- ◆ E.g. Adelman and Robinson 1978

# Linearization

- ◆ Linearize all the equations of the CGE model and then solving the linear approximation by simple matrix inversion.
- ◆ This approach was used in the first applied CGE model formulated by Johansen. 1960.
- ◆ E.g. Dixon et al. 1982.

# Non-linear programming

- ◆ Formulate the CGE model as a non-linear programming model whose solution yields shadow prices that can be interpreted as market prices.
- ◆ E.g. Ginsburgh and Waelbroeck 1981, who also survey the other techniques.

# State of the art

- ◆ Fixed point algorithm died out quickly
  - Empirically very inefficient and impractical
- ◆ The programming approaches little used
- ◆ Linearization (ORANI) and
- ◆ System of non-linear equations have prevailed. CGE model in “levels”

# Solving CGE Models in Levels

- ◆ Tatonnement methods
  - Gauss-Seidel/Jacobi algorithms
- ◆ Jacobian methods
  - Newton-Rapheson
  - M.J.D Powell: Powell algorithm

# GAMS Software

- ◆ General Algebraic Modeling System (GAMS); [WWW.GAMS.COM](http://WWW.GAMS.COM)
- ◆ Designed for modeling linear, nonlinear and mixed integer optimization problems
- ◆ Especially useful with large, complex problems
- ◆ Available for use on PC, workstations, mainframes and supercomputers
- ◆ A large number of solvers for mathematical programming models has been hooked up to GAMS; e.g. MINOS, CONOPT,...

# Typical GAMS Components

## Inputs

- ◆ **Sets**
  - Declaration
  - Assignment of members
- ◆ **Data** (Parameters, Tables, Scalar)
  - Declaration
  - Assignment of values
- ◆ **Variables**
  - Declaration
  - Assignment of type
- ◆ **Assignment of bounds** and/or initial values (optional)
- ◆ **Equations**
  - Declaration
  - Definition
- ◆ **Model and Solve Statements**
- ◆ **Display Statement** (optional)

## Outputs

- ◆ Echo Print
- ◆ Reference Maps
- ◆ Equation Listings
- ◆ Status Reports
- ◆ Results

# GAMS Solvers for CGE models

## ◆ NLP Solvers

- Minos: Stanford
- Conopt: Drud
- Used to solve “square” CGE models or models expressed NLP problems

## ◆ MCP: “Square” solvers

- Miles: Rutherford
- Path: Ferris, Munson, Dirkse

# Square CGE Model

- ◆ The model has one more equation than number of variables.
- ◆ The third identity (Walras' Law) implies that the excess demand equations are functionally dependent.
  - Jacobian is singular.
- Drop one excess demand equation.
  - MCP will solve this model.
  - NLP solvers will also work fine, with arbitrary objective function.

# Walras' Law

- ◆ Walras' Law implies a singular Jacobian of excess demand equations.
  - Standard approach: drop one equation.
- ◆ Alternative: Add a dummy variable
  - “Walras” variable must be zero at solution.
  - Model is square.
- Jacobian is singular at solution.
- NLP and MCP solvers all work.

# NLP Formulation

- ◆ Problem is to find unique feasible basis.
- ◆ Objective function is irrelevant, but judicious choice of objective function may improve solver performance.
- ◆ Feasible to loosen model and optimize.
  - Social welfare function.

# GAMS MCP Solvers

- ◆ Facilitate development of CGE models incorporating complementarity relationships
  - Minimum wage
  - TRQs and import rationing
  - LP technology specification
  - Agricultural policy regimes
  - “Regime shift” macro models

# GAMS NLP Solvers

- ◆ Dynamic optimization: optimal growth models.
  - Rational expectations models.
  - Macro models.
- ◆ Welfare analysis: welfare maximization with policy instruments.

# CGE Models in GAMS

- ◆ Mid 1980s: convert to GAMS
- ◆ See Model Library (under Applied General Equilibrium) in GAMS website for many examples of CGE model.
- ◆ Check Cameroon General Equilibrium Model Using:
  - NLP (Non-linear Programming)
  - CNS (Constrained Nonlinear System)
  - MCP (Mixed Complementarity Problem)
  - MPSGE (Mathematical Programming System for General Equilibrium Analysis)

# Cameroon General Equilibrium Model

- ◆ This general equilibrium model is widely used as a blueprint for new model developments. It follows closely the style and type of model pioneered by Dervis, De Melo and Robinson in the late 1970.
- ◆ Reference: Condon, T, Dahl, H, and Devarajan, S. Implementing a Computable General equilibrium Model on GAMS - The Cameroon Model. Tech. rep., The World Bank, 1987.
- ◆ The original model is formulated as an optimization model, but it is really a square system of nonlinear equations.